Theory of SAR Interferometry and Differential SAR Interferometry

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Multidimensional SAR Systems

SAR Interferometry (InSAR)

Differential SAR Interferometry (DInSAR)
Multidimensional SAR Systems

The multidimensional SAR system acquires \( m \) complex SAR images

Target vector \( \mathbf{k} = [S_1, S_2, \ldots, S_m]^T \)

The properties of the target vector follow from the properties of a single SAR image

- \( \mathbf{k} \) is deterministic for point scatterers. It contains all the necessary information to characterize the scatter.
- \( \mathbf{k} \) is a multidimensional random variable for distributed scatterers due to speckle. A single sample does not characterize the scatterer.

SAR images characterized through second order moments

- Second order moments in multidimensional SAR data are matrix quantities.

Multidimensional SAR Systems

- **SAR Interferometry** (InSAR): \( m=2 \). Topographic information
- **Differential SAR interferometry** (DInSAR): \( m=3 \). Topographic changes information
- **SAR Polarimetry** (PolSAR): \( m=3,4 \). Geometric characterization and classification of the scatterers being imaged
- **Polarimetric SAR interferometry** (PolInSAR): \( m=6,8 \). Study and characterization of volumetric structures
- **SAR Tomography/Multibaseline**: \( m>2 \). Vertical profiling
- **Multitemporal SAR**: \( m>2 \). Change detection and temporal analysis
- **Multifrequency SAR**: \( m>2 \). Characterization of the scatters being imaged

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Important aspects to consider in Multidimensional SAR Imagery

- **Physics**
  - Depending on the configuration of the multidimensional SAR system, information is sensitive to one or several properties of the target being imaged.
  - Data processing, and specially, data estimation can not be done without taking into account the physics behind the scattering process.
  - The most clear example is the number of channels $m$. Represents a clear limitation for multidimensional SAR imagery.

- **Mathematical representation, Statistics**
  - A mathematical description is necessary to systemize data description and understanding.
Interferometric configurations sensitive to the terrain’s topography

Multidimensional SAR Systems

SAR Interferometry

Test Site: Mt. Etna / Sicily, Italy
A complex SAR image $S(x, r) = |S(x, r)| \exp(j \theta(x, r))$

- **Amplitude information**
- **Absolute phase information** composed by two terms $\theta(x, r) = -\frac{4\pi}{\lambda} R + \theta_s$
  - A **deterministic** term due to the 2-way propagation delay
  - A **stochastic** term that depends on the scatterer
- The absolute phase is random and bears no information

SAR Imagery Characteristics

SAR Interferometry

An interferogram is constructed from two SAR images acquired from slightly different spatial positions. Considering a simplified geometry and that scattering is only due to deterministic scatters a **Geometric Approach** is assumed for signal analysis.

$S_1(x, r) = |S_1(x, r)| \exp(j \theta_1(x, r))$

$S_2(x, r) = |S_2(x, r)| \exp(j \theta_2(x, r))$

**Interferogram** $S_1(x, r) S_2^*(x, r)$

**Assumptions**
- Small baseline
- Both images observe the same scatterer
  
  $|S_1(x, r)| \approx |S_2(x, r)|$

  $\theta_1(x, r) \approx \theta_2(x, r)$

**Phase difference**

$$\theta_1(x, r) = -\frac{4\pi}{\lambda} R + \theta_s$$

$$\theta_2(x, r) = -\frac{4\pi}{\lambda} (R + \Delta R) + \theta_s$$

$$\Delta \phi = \theta_2(x, r) - \theta_1(x, r) = -\frac{4\pi}{\lambda} \Delta R$$
The phase difference allows to measure $\Delta R$ with an accuracy in the order of $\lambda$. Wrapped in $2\pi$ due to the circular nature of phase measurements.

The term $\Delta R$ encodes the height information in the interferometric phase $\Delta \theta$ as a function of the system's geometry.

Baseline separation $B$

$$B_x = B \cos(\theta - \alpha)$$

$$B_y = B \sin(\theta - \alpha)$$

Under far field approximation (plane waves):

- $\Delta R^2$ can be neglected.
- $R$ is large (spaceborne SAR system).

First order expansion of the pixel phase:

$$\Delta R \approx B \sin(\theta - \alpha)$$

$$\Delta \phi \approx -\frac{4\pi}{\lambda} B \sin(\theta - \alpha) = -k B \sin(\theta - \alpha)$$
Considering $\Delta \theta$ to be small, i.e., small baseline

$$\Delta(\Delta \phi) = -\frac{4\pi}{\lambda} \Delta(\Delta R)$$

The angle difference $\Delta \theta$ is due to two terms

$$\Delta \phi = \frac{\partial \Delta \phi}{\partial R} \Delta R + \frac{\partial \Delta \phi}{\partial h} \Delta h$$

- **Range variation** (for fixed height)
  $$\Delta \phi \approx \frac{4\pi B_c \Delta r}{\lambda R \tan \theta}$$

- **Height variation** (for fixed range)
  $$\Delta \phi \approx \frac{4\pi B_c \Delta h}{\lambda R \sin \theta}$$

Geometric approach

Due to space diversity InSAR systems are sensitive to topography

- Interferogram phase
- Flat Earth component
- Topographic phase

Height ambiguity (Height coded in a $2\pi$ phase cycle):

$$\Delta h_{2\pi} = \frac{\lambda R \sin \theta}{2B_c}$$

Height sensitivity:

$$\frac{\partial \Delta \phi}{\partial h} = -\frac{4\pi}{\lambda} \frac{B_c}{R \sin \theta}$$

- The larger the baseline the more sensitive the phase with height
- The shorter the wavelength the more sensitive the phase with height
The previous geometric approach only allows to determine the deterministic content of the interferogram

- Simple geometry
- Deterministic scatterers considering only surface scattering

Real scenarios and real InSAR images need from a Stochastic Approach

- Distributed scatterers
- Volume scattering

Distributed scatterers characterization
- Average scattering coeff. $\sigma^2$ (2D)
- Volume scattering coeff. $\sigma_v$ (3D)

\[
R_s(\vec{r},\vec{r}') = E\{\hat{u}(\vec{r})\hat{u}^*(\vec{r}')\} = \sigma_v(\vec{r})\delta(\vec{r} - \vec{r}')
\]

Due to the random component of the reflectivity, interest is focused on the average interferometric response

\[
S_i(x,r)S_i^*(x,r) \quad \Rightarrow \quad E\{S_i(x,r)S_i^*(x,r)\}
\]

The SAR images, after focusing are

\[
S(x_1,\tau_1) = e^{-j2\kappa x_1}\int_{r'} \hat{u}(x',y',z')e^{-j2\kappa r'}h_1(x-x',\tau_1-r')dV''
\]

\[
S(x_2,\tau_2) = e^{-j2\kappa x_2}\int_{r''} \hat{u}_2(x',y',z')e^{-j2\kappa r''}h_2(x-x',\tau_2-r')dV''
\]

Considering
- The scatterer does not change temporarily
- The SAR system is the same in both acquisitions
- Images are co-registered

\[
S(x_1,\tau_1) = e^{-j2\kappa x_1}\int_{r'} \hat{u}(x',y',z')e^{-j2\kappa r'}h(x-x',\tau_1-r')dV''
\]

\[
S(x_2,\tau_2) = e^{-j2\kappa x_2}\int_{r''} \hat{u}_2(x',y',z')e^{-j2\kappa r''}h(x-x',\tau_2-r')dV''
\]
The stochastic or random term represents a very important observable that

- **Degrades** the interferometric information

- **Allows to retrieve the vertical distribution** of scatters in case of volume scattering

\[
E[S_i(x,r)x_i(x,r)] = \exp(-j2(k_1r_1-k_2r_2))\int\sigma_x(r')\exp(-j2(k_1-r_1-r_2))|a(x-x',r_1-r_2)|dV
\]

\(\sigma_x\) is the common volume scattering between both acquisitions
### SAR Interferometry

#### Stochastic approach

<table>
<thead>
<tr>
<th>Absolute «True » Phase</th>
<th>Coherence=1.0</th>
<th>Coherence=0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherence=0.6</td>
<td>Coherence=0.4</td>
<td>Coherence=0.2</td>
</tr>
</tbody>
</table>

**SAR Interferometry**

**Stochastic Approach**

**Necessary conditions** to estimate the expected operator

- **Ergodicity**: The statistical properties of the SAR image (mean, variance, etc...), considered as stochastic process, can be obtained spatially.
- **Stationarity**: The joint probability distribution of the SAR image does not change when shifted in time or space. As a result, parameters such as the mean and variance, if they exist, also do not change over time or position.
  - Wide-sense stationarity
  - Weak-sense stationarity
  - Locally stationary

**Expectation operation obtained through spatial averaging** of the SAR image

\[
E \{ S \} = \frac{1}{\Delta x \Delta y} \int_{-\Delta x/2}^{\Delta x/2} \int_{-\Delta y/2}^{\Delta y/2} S(x, r) \, dx \, dr = \frac{1}{N_x N_y} \sum_{n=1}^{N_y} \sum_{m=1}^{N_x} S(m, n)
\]

Continuous  \quad \rightarrow \quad \text{Discrete}

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Estimation of the complex correlation coefficient through spatial averaging

- Maximum Likelihood Estimator

\[ \rho_{\text{MLT}} = \left| \rho_{\text{MLT}} \right| e^{i\phi} = \frac{\sum_{n=1}^{N} \sum_{m=1}^{M} S_i(m,n) S'_i(m,n)}{\sqrt{\sum_{n=1}^{N} \sum_{m=1}^{M} S_i^2(m,n) \cdot \sum_{n=1}^{N} \sum_{m=1}^{M} S'_i^2(m,n)}} \]

- Phase difference distribution

\[ p_\phi(\phi) = \frac{1}{2\pi\sqrt{\sin\left(\frac{\pi}{M}\right)}} \frac{1}{\beta} \left(1 - |\beta|^2\right)^{\frac{1}{2}} \sin\left(\frac{1}{2}\beta^\prime\right) \]

\[ \beta = |\rho| \cos(\phi - \phi_0) \quad n \text{ number of looks} \]

- The estimated phase is unbiased \( E[\phi] = \Delta \phi_\text{true} \)
- The variance depends on the number of looks \( n \) and on \(|\rho|\)

Coherence distribution

\[ p_{\rho_{\text{MLT}}} \left| \rho_{\text{MLT}} \right| \]
**SAR Interferometry**  
**Stochastic Approach**

**Effect** of the dimension of the spatial averaging

<table>
<thead>
<tr>
<th>$N_x$</th>
<th>$N_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 px</td>
<td>3 px</td>
</tr>
<tr>
<td>5 px</td>
<td>5 px</td>
</tr>
<tr>
<td>7 px</td>
<td>7 px</td>
</tr>
<tr>
<td>9 px</td>
<td>9 px</td>
</tr>
<tr>
<td>21 px</td>
<td>21 px</td>
</tr>
</tbody>
</table>

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**SAR Interferometry**  
**Interferometric Processing Chain**

- Interferometric processor
  - SAR images registration
  - Interferogram generation
  - Coherence
  - Elimination of flat earth component
  - Filtering
  - Phase unwrapping
  - Geocoding

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If the flat earth component is eliminated, topographic phase is obtained.

Topography is related with phase.
Sea and river are totally noisy.
Some noise is observed over the image.

Interferogram phase after removing flat earth term.

Filters, that cancel the frequency bands with no useful signal, are used to reduce the noise.

Resolution is reduced in order to obtain higher phase quality.

- Filters:
  - Linear
  - Non linear
  - Wavelet space
  - Others...
Phase Unwrapping

Phase is wrapped into $2\pi$ intervals. It is necessary to obtain the absolute phase values by means of an unwrapping process.

- Unwrapping methods
  - WLMS (Weighted LMS): Global method with mask
  - RG (Region Growing): Local method
  - Minimum Cost Flow (MCF)
  - Others...

Unwrapped interferogram

Geocoding

Phases must be transformed into geocoded heights over a cartographic grid to obtain the final map.

Methodology:

- Completed unwrapped phase (topography plus flat earth).
- Satellite precise orbits required
- Transformation of phases into WGS-84 X,Y,Z positions.
- Conversion of WGS-84 into UTM coordinates.

Results:

- Error mean: 0.8 meters
- Standard deviation: 10 meters

Geocoded height map
From Interferograms to Maps

SAR Interferometry

Wrapped Phase

Coherence

Unwrapped Phase

DEM (UTM)

Height (m)

0

900

700

600

500

400

300

200

100

Reference DEM (ICC)

InSAR DEM

Error Evaluation

Reference DEM (ICC)

InSAR DEM

Error (ICC-InSAR)

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31

32
Decorrelation is induced by the scatter or by the system

- **Baseline or geometric decorrelation** ($\rho_{\text{spatial}}$), caused by the difference in the incident angles between the two acquisitions
- **Doppler centroid decorrelation** ($\rho_{\text{DC}}$), caused by the differences in the Doppler centroids between both acquisitions
- **Volume decorrelation** ($\rho_{\text{vol}}$), caused by the penetration of the radar wave in the scattering medium
- **Thermal or system noise** ($\rho_{\text{thermal}}$), caused by the characteristics of the system, including gain factors and antenna characteristics
- **Temporal terrain decorrelation** ($\rho_{\text{temporal}}$), caused by the physical changes in the terrain, affecting the scattering characteristics of the surface
- **Processed induced decorrelation** ($\rho_{\text{processing}}$), which results from the chosen algorithms for coregistration and interpolation

The complex correlation coefficient may be decomposed as follows

$$\rho_{\text{tot}} = \rho_{\text{spatial}} \times \rho_{\text{DC}} \times \rho_{\text{vol}} \times \rho_{\text{thermal}} \times \rho_{\text{temporal}} \times \rho_{\text{processing}}$$
Physical Interpretation

Considering the definition of the complex correlation coefficient

\[
\rho = \frac{E \{ S, S' \}}{\sqrt{E \{ |S|^2 \} \cdot E \{ |S'|^2 \}}} = |\rho| \exp(i\varphi)
\]

- **Useful signals**

\[
E \{ S, S' \} = \int \sigma_v |h(x-x', r)|^2 dV' \\
E \{ S, S' \} = \int \sigma_v |h(x-x', r)|^2 dV' + N_1 \\
E \{ S, S' \} = \int \sigma_v |h(x-x', r)|^2 dV' + N_2
\]

- **Complex correlation coefficient** (without interferometric phase)

\[
\rho = \frac{\int \sigma_v (\mathbf{r}) \exp(-j2(\mathbf{k} - \mathbf{k}')) |h(x-x', r)|^2 dV'}{\int \sigma_v |h(x-x', r)|^2 dV'} = \frac{1}{1 + \frac{1}{\text{SNR}}}
\]

**Thermal Decorrelation**

Thermal decorrelation accounts for the decorrelation induced by thermal noise

\[
\rho_{\text{thermal}} = \frac{\int \sigma_v (\mathbf{r}) |h(x-x', r)|^2 dV'}{\int \sigma_v |h(x-x', r)|^2 dV'} = \frac{1}{1 + \frac{1}{\text{SNR}}}
\]

- **Real decorrelation factor**
- **Low reflectivity media (low SNR)** are more affected by thermal decorrelation
Temporal decorrelation accounts for temporal changes between acquisitions:

\[
\rho_{\text{temporal}} = \frac{\iint \sigma_v(\vec{r}) \delta(\vec{x}, \vec{y}) \|\vec{r}'\| \, dV'}{\iint \sigma_v(\vec{r}) \|\vec{x}' - \vec{y}'\| \, dV'}
\]

- **Real** decorrelation factor
- **Does not affect** single pass InSAR acquisitions

Geometric decorrelation accounts for decorrelation due to spatial baseline:

\[
\rho = \frac{\int \sigma_v(\vec{r}) \exp\left(-j2\left(\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r}'\right)\right) \|\vec{r}'\| \, dV'}{\iint \sigma_v(\vec{r}) \|\vec{x}' - \vec{y}'\| \, dV'}
\]

- **Inherent** to InSAR configurations
- **Complex** decorrelation factor

It can be decomposed into two additional terms based on the following assumptions:

- \(\Delta k = k_1' - k_2\)
- \(\Delta \theta = \sin(\Delta \theta) = \frac{B}{r}\)
- \(\sigma_v(\vec{r}) \approx \sigma_v(\vec{z})\)
- \(\exp(-j2(\vec{k}_1 \cdot \vec{r} - \vec{k}_2 \cdot \vec{r}')) = \exp\left(-j2\left(\frac{k \cos \theta B}{r} + \Delta k \sin \theta\right)\vec{y}' + \left(\frac{k \sin \theta B_r}{r} + \Delta k \cos \theta\right)\vec{z}'\right)\)
Range decorrelation accounts for the different look angles

\[
\rho_{\text{range}} = \frac{\int_{-\Delta r}^{\Delta r} \int_{-\Delta y}^{\Delta y} \exp\left(-j2\left(\frac{k \cos \theta B_z + \Delta k \sin \theta}{r} \right) y' \right) h\left(-x', -r'\right) \, dx' \, dy'}{\int_{-\Delta y}^{\Delta y} \int_{-\Delta y}^{\Delta y} h\left(-x', -r'\right) \, dx' \, dy'}
\]

A variation in look angle generates a shift and a stretch of the image terrain spectra, that may be approximated by just a shift in case of systems with small relative bandwidth.

This effect is removed if non-common spectral bandwidth are filtered out by means of Wavenumber Shift Filtering.

Volume decorrelation accounts for the decorrelation induced by a finite distributed scatters in the z dimension

\[
\rho_{\text{vol}} = \frac{\int_{-\Delta z'}^{\Delta z'} \sigma_v(z') \exp\left(-j2\left(\frac{k \sin \theta B_z z'}{r} + \Delta k \cos \theta \right) z' \right) \, dz'}{\int_{-\Delta z'}^{\Delta z'} \sigma_v(z') \, dz'}
\]

- Complex decorrelation factor
- Very important to retrieve information from volumetric scatters (forest, crops, urban, etc...)

Intensity image 1 hour, B=20m
Interferometry generalization based on multiple acquisitions at different spatial baselines

More acquisitions allow to improve the estimation of the topography information:
- Improved filtering
- Improved phase unwrapping
- Analysis of problematic areas
- Topography evolution with differential techniques

How to generalize InSAR?
- Classical InSAR considers $m=2$
- A vector of images, a target vector, is measured instead a single SAR image

Joint representation of interferometric SAR images

\[ k = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_m \end{bmatrix} \]

\[ C_k = E \{ kk^H \} = \begin{bmatrix} E \{ S_1 S_1^H \} & E \{ S_1 S_2^H \} \\ E \{ S_2 S_1^H \} & E \{ S_2 S_2^H \} \end{bmatrix} = \begin{bmatrix} T_1 & \rho \sqrt{T_1 T_2} \\ \rho \sqrt{T_1 T_2} & T_2 \end{bmatrix} \]

The matrix representation allows
- Joint consideration of radiometric (diagonal elements of the matrix) as well as interferometric information (off-diagonal elements of the matrix)
- Easy to extend to multiple SAR acquisitions

Useful information is no longer contained in single SAR images or correlation coefficients but in vectors and matrices. Physical interpretation of the measured SAR data must be considered in terms of these vectors and matrices.

MULTIDIMENSIONAL SAR IMAGERY
SRTM Mission

SIR-C Mission: First Ever Polarimetric Interferometry L/C-Band Quad Pol - April 1994 10 days/October 1994 10 days
*last 3 day repeat pass interferometry (1 day repeat cycle) Time baseline of 1, 2, 3 days and 6 month

SRTM, February 2000

Orbit height: 233 km
Inclination: 57 deg
X-band look angle: 52 deg
X-band swath: 50 km

Single polarization channel
SAR Interferometry

SRTM Mission

New Zealand

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SAR Interferometry

SRTM Mission

The World

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### TanDEM-X quick facts

<table>
<thead>
<tr>
<th>Feature</th>
<th>Details</th>
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<tbody>
<tr>
<td>Launch date</td>
<td>Summer 2010</td>
</tr>
<tr>
<td>Launch site</td>
<td>Baikonur, Kazakhstan</td>
</tr>
<tr>
<td>Launcher</td>
<td>Dnepr-1</td>
</tr>
<tr>
<td>Orbit altitude</td>
<td>514 kilometres</td>
</tr>
<tr>
<td>Orbit inclination</td>
<td>97.4 degrees</td>
</tr>
<tr>
<td>Satellite mass</td>
<td>1330 kilograms</td>
</tr>
<tr>
<td>Satellite dimensions</td>
<td>Height: 5 metres, Diameter: 2.4 metres</td>
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<tr>
<td>Power consumption</td>
<td>730 watts (average)</td>
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<td>Mission operation</td>
<td>German Space Operations Center, Oberpfaffenhofen</td>
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<tr>
<td>Satellite command</td>
<td>Weilheim ground station</td>
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<tr>
<td>Data reception</td>
<td>Ground stations:</td>
</tr>
<tr>
<td></td>
<td>- Inuvik, Canada</td>
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<tr>
<td></td>
<td>- O'Higgins, Antarctic</td>
</tr>
<tr>
<td></td>
<td>- Kiruna, Sweden</td>
</tr>
<tr>
<td>Satellite lifetime</td>
<td>5.5 years (6.5 years for consumables)</td>
</tr>
<tr>
<td>Radar centre frequency</td>
<td>9.65 gigahertz (X-band)</td>
</tr>
</tbody>
</table>

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SAOCOM & SAOCOM-CS main facts

- InSAR and tomographic capabilities
- Fully PolSAR system
- Bistatic capabilities

SAR Interferometry

Differential SAR Interferometry
The Luke Air Force Base, west Phoenix (USA), suffered 5.5% of subsidence due to heavy agricultural pumping, with localized differential subsidence reversing stormwater drainage near the base. As a result, a monsoon rain event in September 1992 flooded the base, resulting in its temporary closure and ~$3 million in damage.

Long term subsidence can be a symptom of a decline in the aquifer's ability to store water and represents a distinct threat to infrastructure. For example, the horizontal strain arising from subsiding ground can result in ground fissures which can further erode with rain. This image shows a fissure, found in the western Phoenix valley (USA), that arose over the period of several days. Fissuring is a very significant concern surrounding overproduction of aquifers and the resulting subsidence.

The vicinity of the closed Hollinger gold mine near Timmins, Ontario (Canada) has experienced extensive subsidence problems in early 2000’s. At Kinross’s closed Hayden Hill gold mine in northeast California (USA), a waste rock dump experienced five notable slope failures during the 1990s.
Effects of Earthquakes

Severe damage to River Road Christchurch (New Zealand) following Feb 22 2011 earthquake

The 1964 Great Alaska Quake caused several downtown areas of Anchorage (USA) to collapse. Here, that 11-foot (3.4 meter) drop — or subsidence — took a line of parked cars with it. Before the event, the sidewalk (left) had been as high as the street (far right).

Introduction

Differential Interferometry

- Detection of earth surface movements using SAR Interferometry.
  - In general those movements are
    - Small, centimetric deformations
    - Low-velocity.
    - Linear and/or non-linear
  - Possible causes: earthquakes, volcanoes, natural subsidence, human activities …,
    but they could imply risk in populated areas
  - Main limitations for DInSAR:
    - Temporal decorrelation: low coherence and low phase quality in a significant amount of pixels due to large temporal baselines
    - Atmospheric artifacts: ghost non-linear deformation and topography

Requirements of a Differential Interferometric Processor:

- Estimation of linear deformation and non-linear deformation
- Estimation of atmospheric artifacts
- Estimation of DEM error
- Flexibility on the number of images required
  - the larger, the better. But reliable results should be expected even with a reduced set
Subsidence can be cataloged based on velocity:

- **Fast Subsidence**
  - The displacement occurs during a short period of time (a few seconds or days).
  - Earthquakes, volcanoes eruptions, etc.
  - Two SAR images with short temporal baseline.

- **Slow subsidence**
  - The displacement occurs during a long period of time (up to some years).
  - Mine explorations, water extractions, oil explorations, etc.
  - Velocity can change with time.
  - Two (or more) SAR images with long temporal baseline.

All these displacements have a magnitude of centimeters.

---

**Differential SAR Interferometry: DInSAR**

What happens if terrain moves between both SAR acquisitions?

\[
\Delta \psi = \Delta \psi_{\text{flat}} + \Delta \psi_{\text{topo}} + \Delta \psi_{\text{mov.}} = \\
= \frac{4\pi}{\lambda} \cdot \frac{B \cdot \Delta r}{r_0 \tan \theta} + \frac{4\pi}{\lambda} \cdot \frac{B \cdot h}{r_0 \sin \theta} + \frac{4\pi}{\lambda} \cdot \Delta \rho
\]

This new component does not depend on spatial baseline.

With ERS/ENVISAT data, a movement phase cycle \((2\pi \text{ radians})\) corresponds to a displacement of \(2.8 \text{ cm}\) in the radar visualization direction.
Short Baseline Method. The easiest scenario. Requires the utilization of an interferometric pair with a baseline as small as possible in order to reduce the effect of the first two terms.

- The flat earth term is mathematically cancelled.
- The smaller the topography the larger the baseline we can afford.

Topography Cancellation with DEM. The most versatile technique. The topographic term is cancelled with a synthetic interferogram generated from a good DEM of the area.

Cancellation with three images. Two of the images (with no movement) are used to generate the DEM. The DEM is used to cancel the topography.

Subsidence problems were reported in an area (El Bages) with small towns. Problems located in particular zones.

The first step consists on the generation of an interferogram using SAR images with the necessary time interval.

- Image 1: 13/06/95
- Image 2: 18/02/98
- 980 days between acquisitions
- Spatial baseline: 36 m
- High temporal decorrelation
- Small town surrounded by forest.
**DInSAR with Topographic Cancellation (II)**

Application of Multilook Complex techniques is very important in noisy images.

Interferogram phase without multilook.  
Interferogram phase with multilook.

\[ \frac{a_1 + jb_1 + a_2 + jb_2 + a_3 + jb_3 + a_4 + jb_4}{4} \]

**DInSAR with Topographic Cancellation (III)**

Urban areas preserve coherence better than non-urban zones

Phase  
Coherence
Inverting the information from a DEM (geocoded heights → phases), the synthetic interferogram related with topography is obtained.

Previous DEM.
Geocoded heights.

Topographic heights + “flat earth”
of the interesting area (1 color cycle = $2\pi$ rad).

DInSAR with Topographic Cancellation (V)

Synthetic topographic phases are subtracted from interferogram phases.

Interferogram phases
(“flat earth” + topo. + mov.).

Topographic phases
+ “flat earth”.

Differential interferogram phases (movement).
Subsidence phases must be unwrapped to obtain the absolute displacement map.

### Unwrapping Method
- **Region-Growing algorithm**
- Selection of pixels with high quality and progressive unwrapping.
- Noisy areas are eliminated.

### Results Validation with DGPS Survey Points

<table>
<thead>
<tr>
<th>Point</th>
<th>Meas. Velocity (cm/year)</th>
<th>DGPS Velocity (cm/year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.1</td>
<td>2.1</td>
</tr>
<tr>
<td>2</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>3</td>
<td>1.3</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Subsidence map of the urban zone (in centimeters)

### Temporal Errors: Tropospheric Effects

The extra travel path due to troposphere can be related with physical magnitudes:

- Ground Pressure ($P$)
- Temperature in Celsius ($t$)
- Partial Water Vapor Pressure ($e$)
- Incidence Angle over the Ground ($\alpha$)

\[
\Delta r = 2.277 \times 10^{-3} P + \left( \frac{1255}{r + 27.15} + 0.005 \right) t - 1.156 \tan^2 \alpha \cos \alpha
\]
Temporal Errors: Ionospheric Effects

The extra travel path due to ionosphere is dependent on the free electron concentration or TEC (Total Electron Contents). TEC values range from 0 up to 30 TECU (TEC Units) depending on:

- Hour of day
- Solar activity
- Latitude ...

The Travelling Ionosphere Disturbances (TID) are regions where the electron concentration highly differs from its surroundings:

- Its extension can be tens of Km
- Its effect can be approximated by a linear term.

Example of differential interferogram of Hector Mine (California, USA) earthquake (October 16th 1999).

Each color cycle corresponds to a displacement of 2.8 cm.  
Time baseline = 35 days
Applications DInSAR: Earthquakes Evaluation

Izmit earthquake (Turkey), August 17th, 1999.
DEM (left) and differential interferogram (right) generated with ERS data.

Applications DInSAR: Volcanoes

Galapagos islands volcanoes.
5 year displacement map (left).
Subsidence sequence, 5.3, 1.1, and 0.5 years respectively (up right).
Applications DInSAR: Subsidence due to Oil Extractions

Subsidence caused by Lost Hills and Belridge oil fields (California).
Subsidence profile of 8 months and 35 days (left).
Subsidence map of 35 days (right).

Reference: Eric J. Fielding, “Rapid subsidence over oil fields measured by SAR interferometry”, Jet Propulsion Laboratory (JPL), Pasadena, California (USA)

Main Limitations on Traditional DInSAR

Temporal Decorrelation. Makes DInSAR measurements unfeasible
- Over vegetated areas. Some interesting regions cannot be monitored
- Where the electromagnetic profiles and/or positions of the scatterers change with time within the resolution cell
- Can be detected with the coherence map

Geometric Decorrelation. Limits the number of image pairs suitable for interferometric applications and prevents one from fully exploiting the data set available
- Short spatial baselines have to be used, except with PS

Atmospheric Inhomogeneities. Create ghost fringes superimposed on each interferogram that can compromise accurate deformation monitoring. Also known as Atmospheric Phase Screen (APS)
- Due to its low-wavenumber spectral behavior cannot be detected and estimated from the coherence map associated with each interferogram.
- It can be detected with redundant interferograms
Use of all the available information in terms of interferograms discarding corrupted information.

**Advanced DInSAR**

Interferogram Set Selection

In order to reduce the huge number of possible interferograms and maximize spatial and temporal coherence, a 3D Delaunay triangulation of the available images is done where the three axes are the temporal baseline, the spatial baseline and the Doppler frequency. Minimization of temporal, geometric and doppler induced decorrelation.
The phase between neighbouring pixels can be decomposed in:

\[
\Delta \phi(x_m, y_m, x_n, y_n, T_i) = \frac{4\pi}{\lambda} T_i \left[ v(x_m, y_m) - v(x_n, y_n) \right] + \frac{4\pi}{\lambda} \frac{b(T_i)}{r(T_i) \sin(\theta)} \left[ \varepsilon(x_m, y_m) - \varepsilon(x_n, y_n) \right] + \frac{2\pi}{v} \Delta z \left[ a_z(x_m, y_m, T_i) - a_z(x_n, y_n, T_i) \right] + \left[ \beta(x_m, y_m, T_i) - \beta(x_n, y_n, T_i) \right] + \left[ \alpha(x_m, y_m, T_i) - \alpha(x_n, y_n, T_i) \right] + \left[ n(x_m, y_m, T_i) - n(x_n, y_n, T_i) \right]
\]

**LINEAR ESTIMATION**
- Linear deformation term
- DEM error
- Doppler difference
- Non-linear deformation
- Atmospheric artefacts
- Decorrelation noise

**NON-LINEAR ESTIMATION**
- DEM errors
- Atmospheric artifacts

**CPT**
- Pixel selection
  - Amplitude vs Coherence
- Phase Linear Terms Estimation
  - DEM error, deformation velocity
- Phase Non Linear Terms Estimation
  - Non-linear deformation, atmospheric artefacts
- Linear velocity deformation maps
- Time deformation series
- Atmospheric artifacts
- DEM errors
**Pixel Selection Criteria**

Distinguish between reliable and not reliable pixel’s phase → Selection

- Coherence stability on the stack of multi-looked interferograms. For this, a certain threshold is defined in order to ensure a minimum phase quality. Those pixels that fulfil a specific threshold value in a certain percentage.

![Graph showing coherence stability](image)

- **COMPROMISE BETWEEN DENSITY AND QUALITY**

  - Certain averaging is performed so spatial resolution is reduced.

- Amplitude dispersion study, $D_A$, on the employed list of radiometrically calibrated SLC images. Pixels with $D_A$ below certain threshold are selected.
  - Spatial resolution is preserved.
  - Theoretically, there is not spatial baseline restrictions.
  - Higher number of images are required for proper pixel selection.

**CPT Linear Terms Estimation**

Triangulation of the selected pixels:

- Delauney triangulation relates all the neighbouring pixels generating non-overlapped triangles.

- Adjustment (arcs), deformation.

- Multi-Layer processing

  - The lower quality pixels benefit from the good pixels.

![Diagram of CPT Linear Terms Estimation](image)
Linear model definition

$$\varphi_{\text{model}}(x_m, y_m, x_n, y_n, T_i) = \frac{4\pi}{\lambda} T_i \Delta \varphi_{\text{model}}(x_m, y_m, x_n, y_n) + \frac{4\pi}{\lambda} r(T_i) \sin(\theta) \Delta \varphi_{\text{model}}(x_m, y_m, x_n, y_n).$$

Linear model adjustment and model coherence function

- The adjustment of the linear model is equivalent to finding the bidimensional frequency of the complex sinusoid derived from the phase term.

$$Y_{\text{model}}(x_m, y_m, x_n, y_n) = \sum_{i=1}^{N} \exp \left[ j \left( \varphi(x_m, y_m, x_n, y_n, T_i) - \varphi_{\text{model}}(x_m, y_m, x_n, y_n, T_i) \right) \right].$$

- The sinusoid is irregularly sampled at the coordinates defined by the available $b(T_i)$ and $T_i$. The incorrect sampling would lead to erroneous estimates.

Quality test and integration

- Links with a model coherence lower than a threshold are rejected.
- The results are integrated with a region growing algorithm.

After removing the linear deformation and DEM error

- Non-linear term of deformation
- Atmospheric artifacts
  
  The different behavior of the atmospheric artifacts in time and space with respect the non-linear movement allows to isolate each one.

  - Atmospheric artifacts: low spatial frequency signal on each interferogram but white process in time.
  - Non-linear deformation: narrower correlation in space and lowpass behavior in time.
The isolation of the atmospheric artifacts and non-linear deformation is implemented with successive filters in both spatial and time domains.

- Coupling between both.

The non-linear deformation is estimated in two steps:
- Spatial Low Resolution (SLR)
- Spatial High Resolution (SHR)

The Singular Value Decomposition (SVD) is used to retrieve the temporal sequence of phases for temporal filtering.
- Least square minimum norm solution when subsets of images are present.

O. Mora, J.J. Mallorqui, A. Broquetas, "Linear and Nonlinear Terrain Deformation Maps from a Reduced Set of Interferometric SAR Images", IEEE TGRS, October 2003

Results with 23 images and 24 interferograms
Campi Flegrei (Naples) Data Set

Results with baselines up to 130 meters.
44 SLC images (1992-2000) and 70 differential interferograms.
Work carried out in collaboration with IREA (Naples)

Results over Campi Flegrei (Naples)
Results over Campi Flegrei (Naples)

Validation Campaign Paris. St. Lazare Railway Station

55 SLC, 216 interf, 4 rg x 20 az.

Linear deformation velocity map of the city of Paris

Deformation evolution
St. Lazare Railway Station

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DEM extraction: Paris

Processing without DEM in order to test the CPT robustness and to extract the original DEM.

- Mean and Stdv of DEM real - DEM calculated

<table>
<thead>
<tr>
<th>Maximum Baseline (m)</th>
<th>Mean(DEM – cal. DEM) (m)</th>
<th>Stdv(DEM – cal. DEM) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>7.59</td>
<td>4.15</td>
</tr>
<tr>
<td>100</td>
<td>4.41</td>
<td>3.37</td>
</tr>
<tr>
<td>150</td>
<td>4.34</td>
<td>3.30</td>
</tr>
</tbody>
</table>

Calculated DEM 50 Bn max - Better extraction with high Bn values
Calculated DEM 150 Bn max - Correct DEM extraction

Non-Linear deformation: Gardanne

Linear deformation map

- Stable area
- Deformation area

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Non-Linear deformation: Gardanne

Deformation area

A

B

D

E

-2.1 Subsidence due to mining activities

0.3 cm/y

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